Maths Primer

You're shortly going on a Lean Six Sigma training programme where we will be covering some statistical tools and techniques. Some people may be feeling a little apprehensive about this; it might be some years since they had to use a calculator in anger. Maths might not have been their favourite subject at school, either – it certainly wasn't mine!

So, the purpose of this document is to try and put minds at ease, and introduce or provide a reminder about some of the concepts and symbols we'll come across.

Let's start by testing your math skills with a little exercise:

1. Think of a whole number greater than one but less than ten
2. Double it
3. Add five
4. Multiply by 50
5. If you have already had your birthday this year, add 1768
6. If you haven't, add 1767
7. Deduct the four digit number for your year of birth
8. You should have a three digit number left

You might have already worked it out, but overleaf, you'll find the significance of the number.

Did you get it right?

The point of the exercise is to try and show that maths can be fun, and it needn't be too difficult either. This last point's especially true when we use software and spreadsheets to work things out for us. And you will, though not in the initial training.

We feel it's important to understand where the answers come from, and so go to first principles to help you understand the context of the numbers.

After the training, it's likely you'll get to use appropriate software to help you with the calculations, as well as receive support from people experienced in all of this.

So, let's look at some basics. We'll be focusing on some of the terms and symbols you'll across and reminding you what they mean.

You've already carried out a calculation involving addition, subtraction and multiplication, and will be totally familiar with the symbols used for these types of calculation:

- $3 + 4 + 6 = 13$
- $5 - 2 - 1 = 2$
- $3 \times 4 \times 5 = 60$
- $72 \div 9 = 8$

But what happens when we introduce brackets into a calculation? In the example below, you can see that we have to separately calculate the answer to the sum in the brackets:

- $3 + (5 \times 6) - 2 = 31$

We don't say $3 + 5 = 8 \times 6 = 48 - 2 = 46$

Something else you may well remember is that when we multiply a minus number by another minus number we get a plus:

- $-3 \times -6 = +18$

We'll come across this concept shortly but first, we need to need to go back to the sixties and talk about squares! Sorry!

- $5^2 = 25$ (This is 5 squared, which means 5 x 5)
- $5^3 = 125$ (Here we're looking at 5 to the power of three, so it's 5 x 5 x5)

We use expressions like 5^3 for convenience, as it's easier than writing out 5 x 5 x 5 in full, and, of course, 2^7, for example, is even more convenient. What do make the answer to that, by the way? See if you get 128.

So, the square of a number is that number times itself. The perfect squares are the squares of the whole numbers, and include 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100, for example.

If we see a square root sign, $\sqrt{}$ then we need to find the number that when multiplied by itself gives you the number that you want the square root of.

So, 5 is the square root of 25, which would be written as $\sqrt{25} = 5$.

Let's look at some familiar words and expressions and clarify their meaning. We'll start with averages.

There are 3 different averages! The mean, median, and mode.
So what's the difference?

Here are the methods to explain and work out the mode, median and mean of this set of numbers:

7, 11, 13, 6, 11, 9, 6, 6, 3

Mode

To work out the mode you find the number that occurs most often.
Tip: To remember mode think of most - the mode is the number that occurs the most.

- Put the numbers in order:
- 3, 6, 6, 6, 7, 9, 11, 11, 13
- Look for the number that occurs most often.
- 3, 6, 6, 6, 7, 9, 11, 11, 13
- 6 occurs most often, so the mode is 6

Median

To work out the median you put the numbers in order and find the middle number. In effect, half of the data will be above the median and half below it.

- Put the numbers in order:
- 3, 6, 6, 6, 7, 9, 11, 11, 13
- Find the middle number:
- 7 is in the middle – it has 4 numbers either side of it, so the median is 7

When there are two middle numbers, the median is halfway between the two.

So, if our numbers were 3, 6, 6, 6, 7, 9, 11, 11, 13, 15 the median would be (7 + 9) ÷ 2 = 8. There are five numbers either side of this median.

Mean

To work out the mean you add all the numbers together and divide the total by the total number of items involved, so:

7 + 11 +13 + 6 + 11 + 9 + 6 + 6 + 3 = 72

There are 9 numbers involved, so, the mean = 72 ÷ 9 = 8

Okay, we've looked at averages, squares and square roots, how multiplying two minuses makes a plus, and how old you are! Well, you're old enough to face standard deviations! Almost.

We need to look at the range first. In doing so, we'll consider the pitfall of averages, and in doing all of this, you'll see the importance of the standard deviation, when we eventually get to it!

It's important to recognise that averages can be very dangerous things. They can disguise what's really happening. We might be reporting that on average it takes 15 minutes to go through passport control at the airport. But to get to that average figure, we'll probably find that some passengers got through in only a few minutes, while some took much longer than the average. And, of course, the customers in any business don't see the average. They see what happens to them!

So, it's important to see the range of performance. Earlier we looked at these numbers and calculated the mean:

7 + 11 +13 + 6 + 11 + 9 + 6 + 6 + 3 = 72

With 9 numbers involved, the mean = 72 ÷ 9 = 8

If we imagine these numbers represent the time it takes to do something, we can say that on average it takes 8 minutes to do it, but our performance ranges from 6 minutes to 13 minutes.

Time to look at the standard deviation.

Standard Deviation

The standard deviation is a more difficult concept than the others we've covered. But it's important you understand what it is and how it will help you.

The standard deviation highlights the variation and range of the data. It can help you find the story behind the numbers.

In fact it's the average distance any one item is from the overall mean average. To try and put that into a context, it can help to learn about what statisticians refer to as normal distribution of data.

A normal distribution of data means that most of the examples in a set of data are close to the "average," while relatively few examples tend to one extreme or the other. Of course, how close, and how far away, will depend on the variation in the data and the range of results.
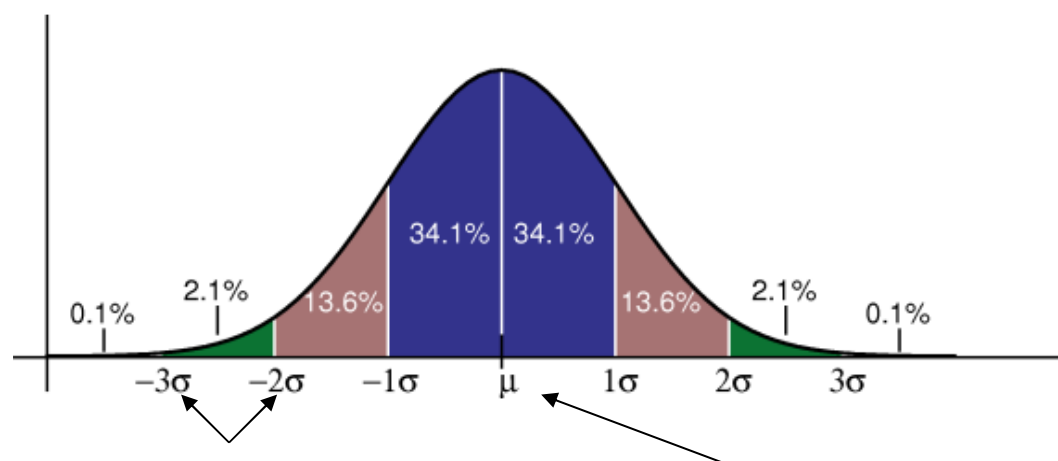
The sigma symbol represents standard deviation; this is the mean of the population. The normal distribution is often referred to as the bell curve because of its shape. And with so many things, you'll find that the more data we have, the more it is likely to be distributed in the shape and proportions shown in the diagram.

It is, however, a theoretical distribution, so we can't expect our real life data and distributions to be exactly like this. Nevertheless, we can broadly expect the following percentages of data to be within so many standard deviations from the mean:

68.26% within plus and minus one standard deviation

95.40% within plus and minus two standard deviations

99.73% within plus and minus three standard deviations

And so on to infinity. We'll look at this in more detail on the course.

Working out the standard deviation is very straightforward with a scientific calculator. And it isn't too difficult doing it with a basic calculator, provided it has the square root function, and provided you don't get phased and frightened by the awful looking formula that is involved.

What we're going to do is work through the steps to find the average distance any one item is from the overall average and once we've done that we'll show you the formula!

So, let's use our set of data again, and make the assumption that this is a representative sample from the entire 'population' of data: 7, 11, 13, 6, 11, 9, 6, 6, 3

We need to work out the mean average, so our first step is to total the numbers:

7 + 11 +13 + 6 + 11 + 9 + 6 + 6 + 3 = 72

With 9 numbers involved, the mean = 72 ÷ 9 = 8

Now we need to calculate the difference between each number and the mean

7 – 8 = -1

11 – 8 = 3

13 – 8 = 5

6 – 8 = -2

11 – 8 = 3

9 –8 = 1

6 - 8 = -2

6 - 8 = -2

3 – 8 = 5

We want to find the average distance any one item is away from the mean, but we can't just add up the differences and divide by the number of items, because the pluses and minuses will balance one another out. So, we square these numbers and in doing so, we'll get rid of all the minuses. Can you remember why?

Hopefully, you remembered the negative times a negative equals a plus rule!

So, let's square the differences:

$-1^2 = 1$

$+3^2 = 9$

$+5^2 = 25$

$-2^2 = 4$

$+3^2 = 9$

$+1^2 = 1$

$-2^2 = 4$

$-2^2 = 4$

$+5^2 = 25$

And now let's total the results:

$1 + 9 + 25 + 4 + 9 + 1 + 4 + 4 + 25 = 82$

Remember, we need to know the average distance any one item is from the overall average, so in theory we should divide this result by 9 (we have 9 numbers in the sample of data). In practice we do something slightly different! Because the nine numbers are a sample, we want to ensure good rigour in our calculation and for reasons outside the scope of this primer, we actually divide by $9 - 1 = 8$ I know this will seem strange, but it just gives us a more statistically sound number. If we had taken a different sample we would have had slightly different results , so this adjustment tries to make allowance for it.

Remarkably, if our sample size is 30 or more, it really makes no difference! So:

$82 \div 8 = 10.25$

Of course, this answer is still in 'square units' – and we tend not to work in square minutes, for example, so we need to get back to our original units. To do that, we need to take the square root of 10.25

$\sqrt{10.25} = 3.20$

So, the standard deviation of our data set is 3.20. But what does this really mean?

If we go back to the diagram of normal distribution, we would expect that approximately two thirds of our data will be within plus or minus one standard deviation from the mean. So, in our example, with a mean of 8, and a standard deviation of 3.20, we should find around two-thirds of our bigger population data between 4.80 and 11.20.

In our sample, seven of nine data points lie in that range (incidentally, the 7 of 9 reference is nothing to do with Star Trek Voyager). Now 7 of 9 is a little more than two thirds – it's nearly 78% of the data, but, of course, we're looking at a small sample from a larger population of data.

If we took all the data from our population the distribution would look more 'normal' and we'd find the following picture emerging:

68.26% within plus and minus one standard deviation

95.40% within plus and minus two standard deviations

99.73% within plus and minus three standard deviations

Oh, I promised to show you the formula, but you probably won't like it!

The Standard Deviation for a sample.   And for a population:

$$\sqrt{\frac{\Sigma \ (\ X_i - \overline{X})^2}{n\text{-}1}} \qquad\qquad \sqrt{\frac{\Sigma \ (\ X_i - \overline{X})^2}{n}}$$

Σ is 'the sum of' or total; x bar (the x with a little bar above it), is 'the mean'.

The X_i represents all of the values of X. So, in our example, there were 9 data items; and n is the number of data items. Again, in our example the n - 1 was 9 –1, (the nine numbers in our data set minus one, which gave us eight).

As you can see, if you had all the data from a population, rather than a sample, then you would divide by n, whatever that number of data items in the population was.

Please remember that despite the formula looking difficult, you just worked out a standard deviation, and now you're going to do another one. What's the standard deviation of 1, 2, 3?

Hopefully, you made it 1!

Before saying goodbye to the Standard Deviation, it's worth reflecting on what it tells us. It measures the variation in the data and shows the range or spread of the results. If the standard deviation is a small number, you will have a tight distribution; if the standard deviation is a high number, it will be a wider distribution. And you can see the effect of this in the examples that follow:

The distribution on the left is tighter, whereas the one on the right shows more variation. The standard deviation for the tighter distribution will be a smaller number than the standard deviation for the distribution on the right. This might be helpful information to you when analysing the results from different teams or locations, for example.
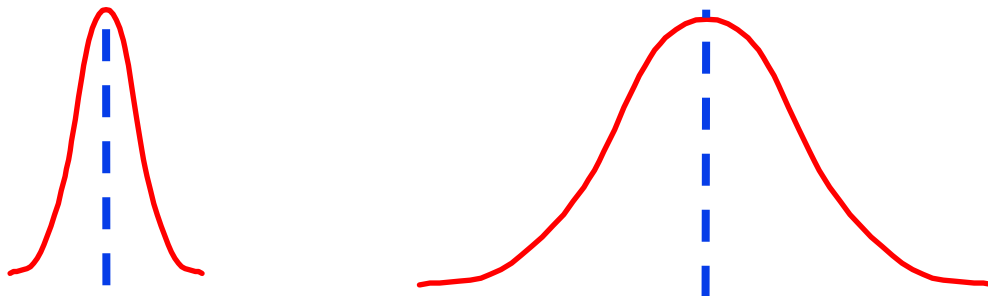
Either way, the data itself will still broadly follow the pattern we referred to earlier, with approximately two-thirds of it plus and minus one standard deviation away from the mean. Around 95% will be within plus and minus two standard deviations, and so on.

As promised, we'll look at this in more detail on the training.

In covering standard deviation, we began to refer to percentages, and decimals came into play, too. We'll be using both in the training, so it's worth a few simple reminders about them.

If we have process 75 items and find that 15 are wrong, what's the percentage of errors? Let's go through the steps to work out the percentage:

1. Take the number of errors and divide this number by the total number of items processed.
2. Multiply your answer by 100 and you have the percentage error rate

So, in our example

1. (15 ÷ 75) = 0.20
2. 0.20 x 100 = 20%

Can you see that 0.20 is another way of expressing 20%; and 0.5 is the same as 50%? O.O5 is the same as 5% and so on.

Our example deliberately created an answer of 20% because it links so well to something else we'll be looking at in the training – the 80/20 rule.

In 1906, Italian economist Vilfredo Pareto created a mathematical formula to describe the unequal distribution of wealth in his country, observing that twenty percent of the people owned eighty percent of the wealth. After Pareto made his observation and created his formula, many others observed similar phenomena in their own areas of expertise.

Quality Management pioneer, Dr. Joseph Juran, working in the US in the 1930s and 40s recognised a universal principle he called the "vital few and trivial many."

The 80/20 Rule means that in anything a few (20 percent) are vital and many (80 percent) are trivial. In Pareto's case it meant 20 percent of the people owned 80 percent of the wealth. In Juran's initial work he identified 20 percent of the defects causing 80 percent of the problems. Project Managers know that 20 percent of the work (the first 10 percent and the last 10 percent) consume 80 percent of your time and resources. You can apply the 80/20 Rule to almost anything, from the science of management to the physical world.

Pareto's Principle, the 80/20 Rule, should serve as a daily reminder to focus 80 percent of your time and energy on the 20 percent of you work that is really important. Don't just "work smart", work smart on the right things.

We look forward to seeing you on the training programme and hope this primer has been helpful preparation. In case you're wondering, we'll look at what we mean by sigma, and in particular, Six Sigma, during the training, but we need to have something mathematical for you to look forward to!